Sidelobe Reduction of a Scanned Circular Array using Particle Swarm Optimization Algorithm

D.Mandal[1], K. S. Kola[2], A.K.Bhattacharjee[2]
[1]Department of Electronics and Communication Engineering
Bengal College of Engineering and Technology,
[2]Department of Electronics and Communication Engineering
National Institute of Technology
Durgapur, India

deb.mandal22@gmail.com[1], kalyankola12@gmail.com[2], akbece12@yahoo.com[3]

ABSTRACT
In this paper the authors propose a pattern synthesis method to reduce the peak side lobe level (peak SLL) of a circular array scanned to two particular directions in the X-Z plane. Synthesized beam-patterns with lower peak SLL are achieved by finding out 4 bit optimum amplitudes of the array elements using Particle swarm optimization (PSO) algorithm. Two different cases have been considered. In the first case, peak SLL of the scanned array has been reduced considering HPBW and FNBW constrains. In the second case, peak SLL has been reduced without HPBW and FNBW constrains.

KEYWORDS: *circular array, first null beam width (FNBW), half power beam width (HPBW), particle swarm optimization (PSO), peak side lobe level, scanned beam*

I. INTRODUCTION

The circular array, in which the elements are placed in a circular ring, is an array configuration of very practical interest. Its applications span radio direction finding, air and space navigation, underground propagation, radar sonar, and many other communication based systems [1-2]. The array elements are placed on the X-Y plane. The peak of the main beam of an antenna array can be directed to any desired direction with the help of progressive phase difference between the elements, commonly known as scanning of the array [1-2]. The first null beam width (FNBW) and half power beam width (HPBW) of a uniformly excited array increases significantly when the array is scanned off the broadside direction. Scanning in the circular arrays directed the main beam towards the desired direction without changing the array position. To radiate the maximum power in the desired direction, minimization of peak side lobe level (peak SLL) is required.

N. Goto *et. al* reduced the side lobe level of circular array with constant amplitude by adjusting the excitation phase distribution of the array element [3]. The optimum distributions of phases were determined by linear programming while satisfying a predefined constrains [3]. F. Watanabe *et.al* minimized peak SLL of a circular array of monopoles by finding out an optimum set of excitation phase of the elements using method of approximation programming (MAP) [4]. The mutual couplings between the array elements were considered and the current distributions of the elements excited by unity voltage were analyzed by Galerkin’s method [4]. M. Shihab *et.al* considered optimum elements weight along with optimum separations between the elements of circular arrays with different no of elements to reduce the maximum SLL of the arrays [5]. The optimum sets of parameters were determined by PSO [5] and were shown as a better alternative than GA to reduce the max SLL for the circular arrays. G. G. Roy *et.al*, reduce the SLL of circular array for specified value of first null beam width (FNBW) by finding out optimum set of normalized amplitudes and optimum set of inter element distances for the array using invasive weed optimization (IWO) algorithm and was proven superior than GA,PSO and DE [6].

In this paper the peak SLL of a circular array, scanned to two different angle of $\left(\theta_0 = 30^\circ, \varphi_0 = 0^\circ\right)$ and $\left(\theta_0 = 45^\circ, \varphi_0 = 0^\circ\right)$ are reduced by finding out optimum set of elements amplitudes using Particle Swarm Optimization (PSO) algorithm [7-14]. Two different cases are considered. In the first case, the peak SLL are reduced while keeping HPBW and FNBW of the optimized beam patterns less than or equal to the HPBW and FNBW of same array with uniform excitation and same scanning angles. In the second case, peak SLL are reduced without any HPBW and FNBW constrains.

II. PROBLEM FORMULATION

The far field pattern of circular array of isotropic elements shown in Figure 1 on X-Y plane scanned to a specified angle can be defined as[1-2]:

$$AF(\theta,\varphi) = \sum_{n=1}^{N} I_n e^{jkr[\sin\theta\cos(\varphi-\varphi_n)-\sin\theta_0\cos(\varphi_0-\varphi_n)]} \quad (1)$$

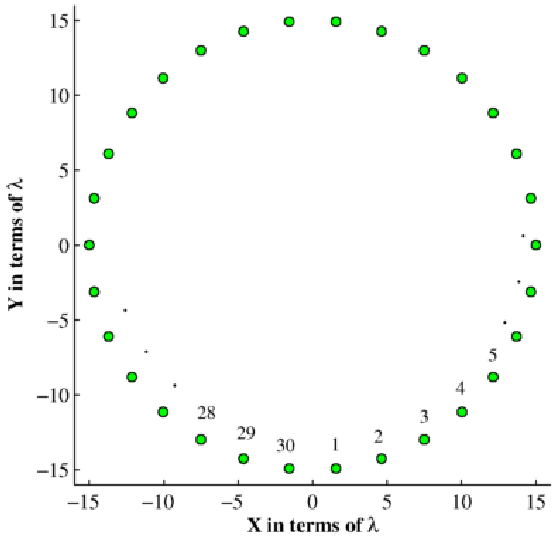

Figure 1. Geometry of a circular array of 30 isotropic elements.

where, $N =$ number of isotropic element in the array; $I_n =$ excitation amplitude of n-th element; $r =$ radius of the circular array; $k = 2\pi/\lambda$, represents wave number; $\theta,\varphi =$ polar and azimuth angle; $\varphi_n = 2\pi(\frac{n}{N})$ is the angular location of the n-th element and (θ_0, φ_0) steering angle.

Normalized absolute power pattern, $P(\theta,\varphi)$ in dB can be expressed as follows:

$$P(\theta,\varphi) = 10\log_{10}\left[\frac{|AF(\theta,\varphi)|}{|AF(\theta,\varphi)|_{\max}}\right]^2 = 20\log_{10}\left[\frac{|AF(\theta,\varphi)|}{|AF(\theta,\varphi)|_{\max}}\right] \quad (2)$$

Fitness functions for two different cases are as follows:

$$Fitness1 = k_1 \max SLL + k_2(FNBW_o - FNBW_d)H(T_1) + k_3(HPBW_o - HPBW_d)H(T_2) \quad (3)$$

$$Fitness2 = \max SLL \quad (4)$$

Equation (3) and Equation (4) are reduced individually using PSO for optimum synthesis of the array, where max SLL is the value of maximum side lobe level, $FNBW_o$, $FNBW_d$ are obtained and desired value of first null beam width and $HPBW_o, HPBW_d$ are obtained and desired value of half power beam width respectively. k_1, k_2 and k_3 are the weighting coefficients to control the relative importance given to each term of Equation (3). Equation (4) is for reducing peak SLL without $FNBW$ and $HPBW$ constrains.

$H(T_1)$ and $H(T_2)$ are Heaviside step functions defined as follows:

$$T_1 = (FNBW_o - FNBW_d)$$

$$H(T_1) = \begin{cases} 0, & if\ T_1 < 0 \\ 1, & if\ T_1 \geq 0 \end{cases} \quad (5)$$

$$T_2 = (HPBW_o - HPBW_d)$$

$$H(T_2) = \begin{cases} 0, & if\ T_2 < 0 \\ 1, & if\ T_2 \geq 0 \end{cases} \quad (6)$$

III. PARTICLE SWARM OPTIMIZATION ALGORITHM

Particle Swarm Optimization (PSO) algorithm is a population based stochastic optimization tool inspired by social behavior of bird flock, fish school etc. as developed by Kennedy and Eberhart in 1995 [7]. It exploits a population of potential solutions to probe the search space concurrently. In PSO, the population is called the swarm and its individuals are called the particles. Each particle is associated both with a position and velocity in the search space. The swarm is defined as a set of *N* particles as: $s = \{x_1, x_2, ..., x_n\}$ and the position and velocity of a particle in *n*-dimensional problem is defined as [7-14]:

$x_i = (x_{i1}, x_{i2}, ...x_{in})^T \in A$, for i=1,2,…,*N* and *A* is the search space. (7)

$v_i = (v_{i1}, v_{i2}, ..., v_{in})^T$ for i=1,2,…,*N*. (8)

The velocity and the positions of the particles are changes with iterations and these changes are dependent on the performance of the particles itself and that of the other particles. The algorithm can be summarized as follows:

Step 1: Initialization

Randomly initialize the position and the velocity of *N* particles at iteration *t* using the following equations:

$x_{ij}(t) = x_{\min} + rand(x_{\max} - x_{\min})$ for *i=1,2,…,N* and *j=1,2,…,n*. (9)

$v_{ij}(t) = (-\frac{x_{\max} - x_{\min}}{2} + rand(x_{\max} - x_{\min}))$ for *i=1,2,…,N* and *j=1,2,…,n*. (10)

where, x_{max} and x_{min} are the upper and the lower bound of the problem and *rand* is a uniform random variable within [0,1].

Step 2: Fitness evolution at current iteration:

Compute the fitness for each particle in the swarm at current generation,
i.e., evaluate $f(x_i)$ for *i= 1,2,…,N*

Step 3: Compute $pbest_i$ and $gbest$

Compute the best previous position of the i-th particle $pbest_i$ and global best position $gbest$ among all the particles in the swarm at current iteration using the following equations [7-14]:

$$pbest_i(t) = \arg\min_t f_i(t) \quad (11)$$

$$gbest(t) = \arg\min_i f(pbest_i(t)) \quad (12)$$

Step 4: Update particles position and velocity

Update the velocity and the position of the particles at iteration *t* using the following equations:

$$v_{ij}(t+1) = wv_{ij}(t) + c_1 rand_1(pbest_{ij}(t) - x_{ij}(t)) + c_2 rand_2(gbest_j(t) - x_{ij}(t)) \quad (13)$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \quad (14)$$

for $i=1,2,\ldots,N$ and $j=1,2,\ldots,n$. *rand1* and *rand2* are two uniformly distributed random variables between [0,1] and c_1 and c_2 weighting factors known as cognitive and social parameter. w is called inertia weight. It is defined as [7-14]:

$$w = w_{\max} - \left(\frac{w_{\max} - w_{\min}}{T}\right)t \quad (15)$$

Where T is the maximum iteration, and $w_{\max}$, $w_{\min}$ are set to 0.9 and 0.4.

Restrict the velocity of the particles such that $\left|v_{ij(t+1)}\right| \le v_{\max}$, for $i=1,2,\ldots,N$ and $j=1,2,\ldots,n$.

This is known as velocity clamping and is implemented using the following equation [7-14]:

$$v_{ij}(t+1) = \begin{cases} v_{\max}, & if \quad v_{ij}(t+1) > v_{\max,} \\ -v_{\max} & if \quad v_{ij}(t+1) < -v_{\max} \end{cases} \quad (16)$$

Step 5: Repeat from step 2-5 until iterations reaches its maximum limit. Return $gbest$ as the final result.

IV. RESULTS AND DISCUSSION

A circular array of 30 isotropic elements is considered. The peak SLL, HPBW and FNBW of the uniformly excited array scanned to two different directions are shown in Table 1. From Table 1, it can be seen that when the array is uniformly excited and scanned in the direction $\left(\theta_0 = 30^\circ, \varphi_0 = 0^\circ\right)$, the value of peak SLL, HPBW and FNBW of its obtained beam pattern are

Table 1. Results for uniform excited array

Parameters	**Scanned angle**	
	$\left(\theta_0 = 30^\circ, \varphi_0 = 0^\circ\right)$	$\left(\theta_0 = 45^\circ, \varphi_0 = 0^\circ\right)$
peak SLL (dB)	-7.89	-7.89
HPBW (degree)	10.00	12.30
FNBW (degree)	21.40	26.90

Table 2. Desired and obtained results for two different cases

Design Parameters		**Scanned angle for case I**		**Scanned angle for case II**	
		$(\Theta_0=30^0, \varphi_0=0)$	$(\Theta_0=45^0, \varphi_0=0)$	$(\Theta_0=30^0, \varphi_0=0)$	$(\Theta_0=45^0, \varphi_0=0)$
Peak SLL		-9.4198	-9.7016	-19.2262	-16.4751
HPBW (degree)	Desired	8.60	8.60	10.00	12.30
	Obtained	10.00	12.30	13.20	15.50
FNBW (degree)	Desired	18.40	18.40	21.40	26.90
	Obtained	21.40	26.70	31.10	34.90

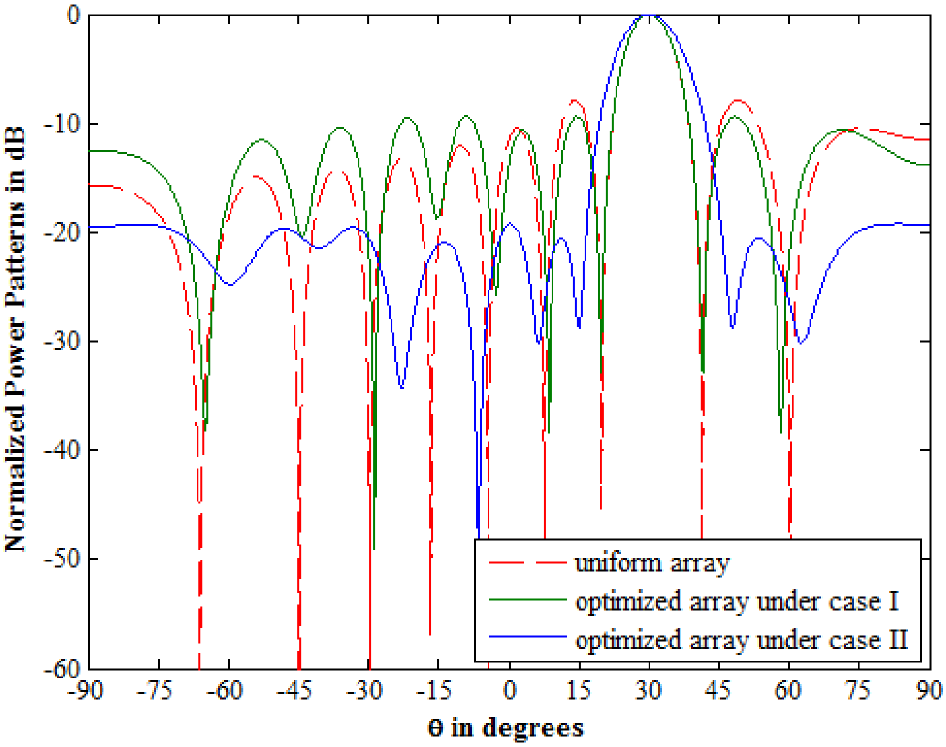

Figure 2. Far field patterns of the array scanned to $\left(\theta_0 = 30^\circ, \varphi_0 = 0^\circ\right)$ for case I and Case II

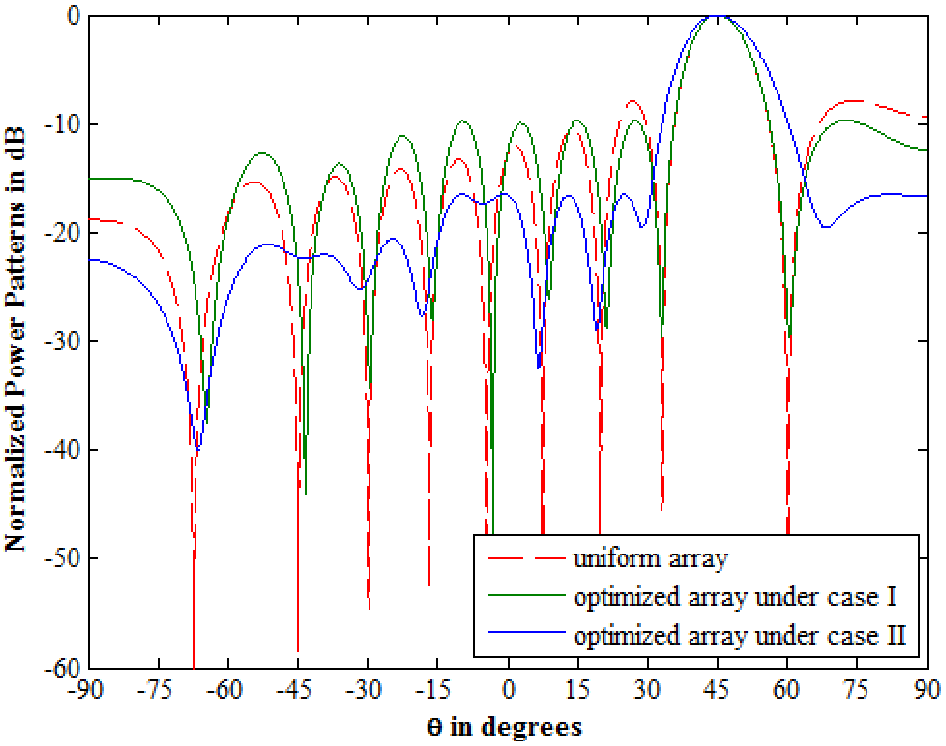

Figure 3. Far field patterns of the array scanned to $\left(\theta_0 = 45^\circ, \varphi_0 = 0^\circ\right)$ for case I and Case II

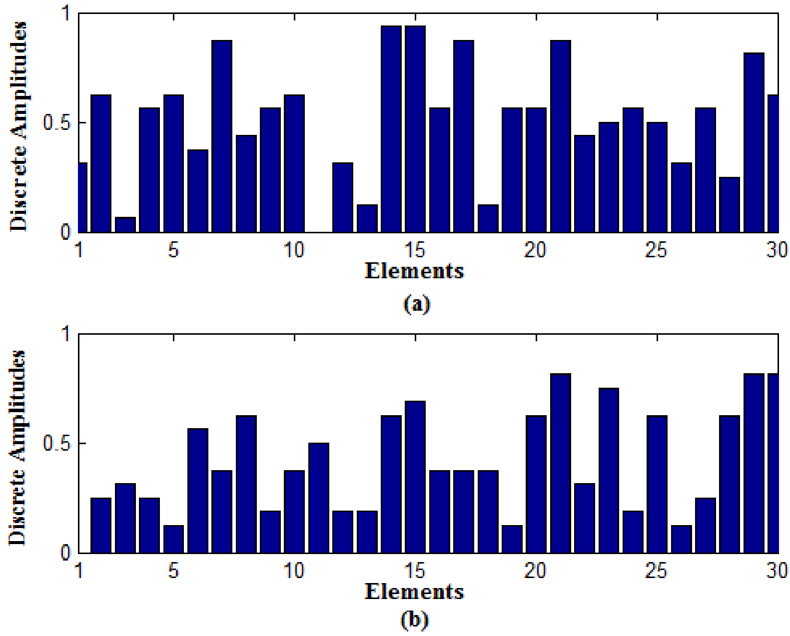

Figure 4. Discrete Amplitudes for case I (a) $\left(\theta_0 = 30^\circ, \varphi_0 = 0^\circ\right)$ (b) $\left(\theta_0 = 45^\circ, \varphi_0 = 0^\circ\right)$

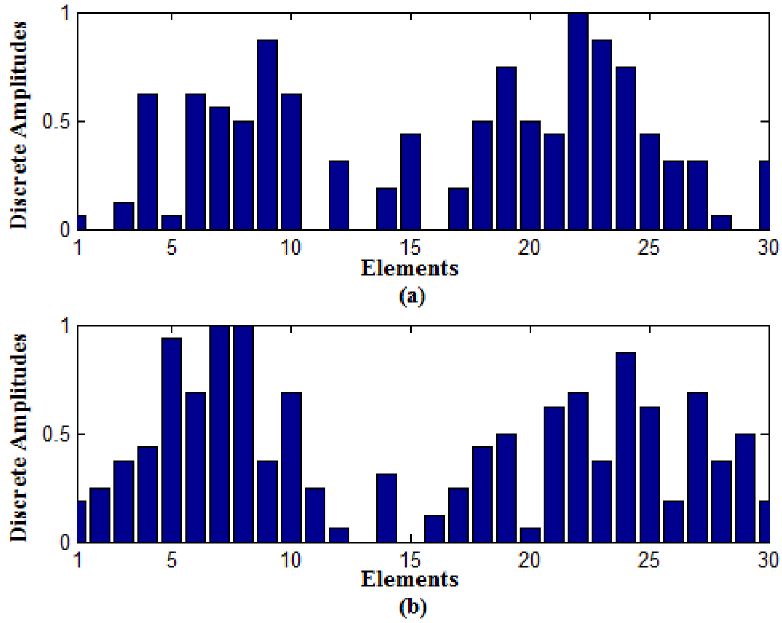

Figure 5. Discrete Amplitudes for case II (a) $\left(\theta_0 = 30^\circ, \varphi_0 = 0^\circ\right)$ (b) $\left(\theta_0 = 45^\circ, \varphi_0 = 0^\circ\right)$

-7.89 dB, 10.00 degree and 21.40 degree, whereas when it is scanned in $\left(\theta_0 = 45^\circ, \varphi_0 = 0^\circ\right)$, the obtained values of the same parameters are -7.89 dB, 12.30 degree and 26.90 degree. A comparison between the reductions of peak SLL once keeping FNBW and HPBW fixed and once by without

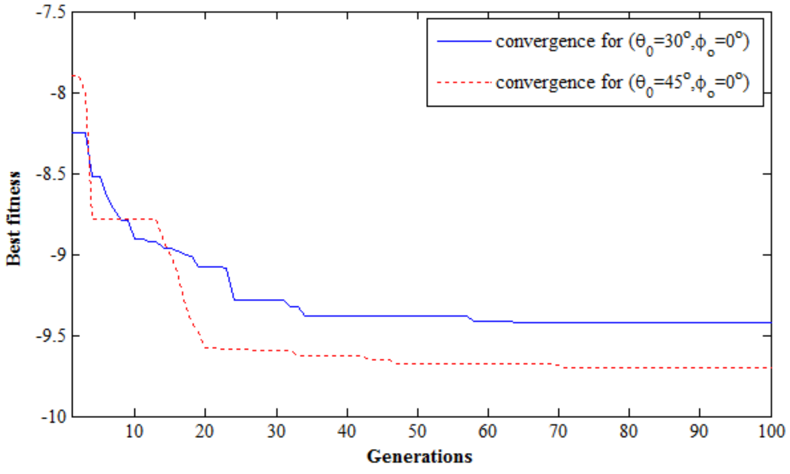

Figure 6. convergence curve for case I.

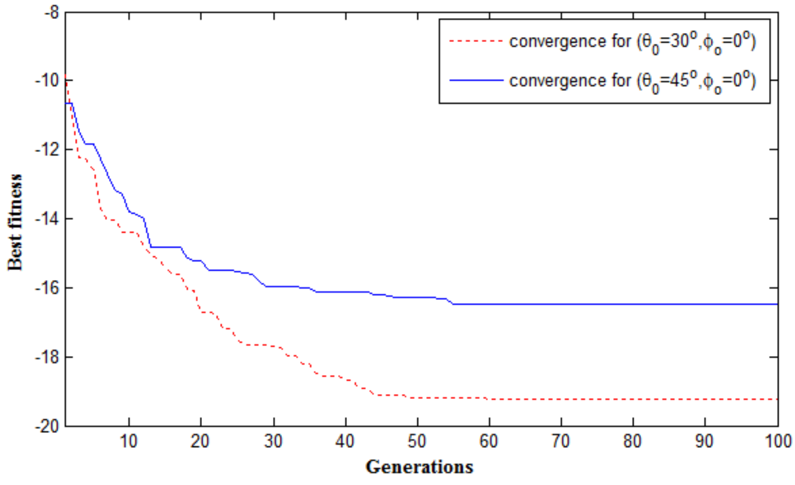

Figure 7. convergence curve for case II

those constrains at two different scanning angles are presented in Table 2. For the first case, the obtained values of peak SLL for two different scanning directions are -9.41dB and -9.70dB respectively. The obtained values of HPBW and FNBW of the obtained beam pattern scan to $\left(\theta_0 = 30^\circ, \varphi_0 = 0^\circ\right)$ are 10.00 degree and 21.40 degree corresponding to their desired values of 8.60 degree and 18.40 degree respectively. Similarly, under same case when the beam scanned to a direction $\left(\theta_0 = 45^\circ, \varphi_0 = 0^\circ\right)$ the obtained values of HPBW and FNBW are 12.30 degree and 26.70 degree, corresponding to the same desired values mentioned for the earlier scanning angle. The second case depict that the obtained value of peak SLL for two different scanning angle are
-19.22dB and -16.4751dB, are much lesser than the first case. Whereas the obtained values of HPBW and FNBW for two different scanning angles under this second case are 13.20 degree, 15.50 degree and 31.10 degree, 34.90 degree. In both the cases particle swarm optimization is used for finding out optimum sets of discrete amplitudes to minimise the SLL. The results clearly indicate the increment of FNBW and HPBW causes lower side lobe level for cases II. The optimum amplitudes of the array elements computed using PSO for both the cases are shown in Figure 4 and Figure 5 respectively. The convergence characteristics of PSO algorithm between the two different scanning angles for both the cases are shown in Figure 6 and Figure 7 respectively. Better convergence of scanning angle

$\left(\theta_0 = 45^\circ, \varphi_0 = 0^\circ\right)$ over the scanning angle $\left(\theta_0 = 30^\circ, \varphi_0 = 0^\circ\right)$ for both the cases of the design problem is clearly indicated in Figure 6 and Figure 7.

V. CONCLUSIONS

The presented method shows a technique to reduce peak SLL of a circular array whose beam patterns are scanned to two different angles in the X-Z plane with the help of progressive phases among the elements. The method incorporate PSO algorithm to find out optimum amplitudes of the array elements to obtain the desired beam pattern. To ease the design of the feed network, the presented synthesis problem introduce 4-bit discrete amplitude among the array elements, which will be helpful for designing the feed network and also minimize the effect of mutual coupling among the array elements.

The presented method can also be extended for other types of array configuration.

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