Modeling and Analysis of Six-phase Self-excited Induction Generators for Wind Energy Conversion

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ABSTRACT

A simple mathematical model is proposed to compute the steady-state performance of six-phase self-excited induction generators. A mathematical model is formed directly from the equivalent circuit of six-phase self-excited induction generator by nodal admittance method. The proposed model completely eliminates the tedious work involved in the existing models. Genetic algorithm is applied to solve the proposed model. The capacitance requirements to maintain constant voltage for various configurations are tabulated. The detailed winding scheme of six-phase self-excited induction generator is also presented. The analytical results are found to be in close agreement with the experimental results.

Keywords: Self-excited induction generator, Genetic algorithm, Renewable energy sources & Wind energy conversion

I. INTRODUCTION

The energy demand increases rapidly during the last few decades, due to the depletion of the world fossil fuel supplies. Thus, the use of renewable energy sources becomes essential and therefore, the study of self-excited induction generator (SEIG) has regained importance as it is particularly suitable [1] for wind and small hydro power plants. Even though three-phase induction generators are used for this purpose, many research articles have been reported [2-7] on multiphase (more than three phase) induction machine due to their advantages like higher power rating and improved reliability.

Regarding the mathematical modelling of six-phase self-excited induction generators (SPSEIG), the concept of three-phase and single-phase self-excited induction generator modeling [8-13] can be utilized. However, they need separation of the real and imaginary components of the complex impedance or admittance of the equivalent circuit to derive specific models which are tedious. The author made an attempt for the first time to overcome the complication of three-phase and single-phase SEIG models [14-15] by introducing the concept of graph theory which avoids the lengthy and tedious mathematical derivations of nonlinear equations.

In the present paper, the author has developed a further simplified mathematical model of SPSEIG in matrix form using nodal admittance method based on inspection. In the proposed model, the nodal admittance matrix can be formed directly from the equivalent circuit of SPSEIG [6] by inspection rather than deriving it from the concept of graph theory [14-15]. Moreover, this model is also flexible such that any equivalent circuit elements can be easily included or eliminated. Genetic algorithm (GA) is proposed to determine the capacitive VAr requirement of SPSEIG. Experiments are conducted on a prototype SPSEIG and the experimental results are found to be good agreement with the analytical results.

This paper is organized as follows. Section 2 talks about proposed mathematical model, section 3 describes the performance evaluation of SPSEIG using genetic algorithm. The experimental set up and machine parameters are given in section 4. Section 5 presents the capacitance and VAr...
requirements for different topologies of SPSEIG. This paper ends with our conclusions in section 6. The winding scheme and the photograph of the SPSEIG are presented in the Appendix.

II. MATHEMATICAL MODEL

The formulation of a suitable mathematical model is the first step in the analysis of a SPSEIG. The model must describe the characteristics of the individual components of the SPSEIG as well as the relations that govern the interconnections of these elements. Therefore, a mathematical model of a SPSEIG using nodal analysis based on inspection is developed from the equivalent circuit of the generator. The developed model results in a matrix form that proves convenient for computer simulations.

The steady state equivalent circuit of six-phase self-excited induction generator [6] is shown in Figure 1. The equivalent circuit is valid for any per unit speed \( \bar{v} \). The various elements of equivalent circuit are given below.

\[
\begin{align*}
Y_1 &= 1 / \{R_r / (F - \bar{v}) + j X_r\}; \quad Y_2 = 1 / \{jX_M\}; \quad Y_3 = 1 / \{jX_{lm}\}; \\
Y_4 &= 1 / \{R_{s2} / F + j X_{s2}\}; \quad Y_5 = 1 / \{R_{s1} / F + j X_{s1}\}; \quad Y_6 = 1 / \{-jX_{sh2} / F^2\}; \\
Y_7 &= 1 / \{R_{L2} / F\}; \quad Y_8 = 1 / \{-jX_{sh1} / F^2\}; \quad Y_9 = 1 / \{R_{L1} / F\};
\end{align*}
\]

Figure 1. Per phase equivalent circuit representation of six-phase self-excited induction generator.

The matrix equation based on nodal admittance method for the equivalent circuit can be expressed as

\[
[Y][V] = [I_s]
\]  

(1)

Where \([Y]\) is the nodal admittance matrix, 

\([V]\) is the node voltage matrix, and \([I_s]\) is the source current matrix.

The \([Y]\) matrix can be formulated directly from the equivalent circuit (Figure 1) using nodal admittance method based on inspection [16] as

\[
[Y] = \begin{bmatrix}
Y_1 + Y_2 + Y_3 & -Y_3 & 0 & 0 \\
-Y_3 & Y_3 + Y_4 + Y_5 & -Y_5 & -Y_4 \\
0 & -Y_5 & Y_5 + Y_8 + Y_9 & 0 \\
0 & -Y_4 & 0 & Y_4 + Y_6 + Y_7
\end{bmatrix}
\]  

(2)

where

\(Y_i = \sum\) Admittance of the branches connected to \(i^{th}\) node
Y_{ij} = - \sum \text{Admittance of the branches connected between } i^{th} \text{ node and } j^{th} \text{ node}

Since, the equivalent circuit does not contain any current sources, \([I_s] = [0]\) and hence Eq. (1) is reduced as
\([Y] [V] = 0\) \hspace{1cm} (3)

For successful voltage build up, \([V] \neq 0\) and therefore from Eq. (3), \([Y]\) should be a singular matrix i.e., \(\det [Y] = 0\). It implies that both the real and the imaginary components of \(\det [Y]\) should be independently zero. Therefore to obtain required parameter which results \(\det [Y] = 0\), genetic algorithm based approach is implemented.

### III. Performance Evaluation of SPSEIG using Genetic Algorithm

Application of genetic algorithm [17] to obtain \(\det [Y] = 0\), which provides solution for unknown quantities, is illustrated in Figure 2. The objective function whose value is to be minimized is given by Eq. (4).

\[ g (F, X_M \text{ or } X_{sh}) = \text{abs}\{\text{real}(\det [Y])\} + \text{abs}\{\text{imag}(\det [Y])\} \] \hspace{1cm} (4)

In many optimization problems to obtain initial estimates suitably, certain trials may be required. However, in the present problem of the SPSEIG, it is easy to give the range for the unknown variables \(F\) and \(X_M\) or \(X_{sh}\) because in well-designed self-excited induction generators, it is known that the slip
\{(F - \upsilon)/F\} is small and operation of the machine is only in the saturated region of the magnetization characteristics. So, the ranges for \(F\) can be given as 0.8 to 0.999 times the value of \(\upsilon\) and for \(X_M\) as 25\% to 100\% of critical magnetizing reactance \(X_{MO}\). Similarly for \(X_{sh}\), the same range 25\% to 100\% of \(C_{MAX}\) can be used, where \(C_{MAX}\) is the maximum capacitance required under any conditions. Thus, starting from such initial estimates, the final value of \(F\) and \(X_M\) or \(X_{sh}\) is obtained through GA. The air gap voltage \(V_g\) can be determined from the magnetization characteristics corresponding to \(X_M\), as described in Section 4. Once the air gap voltage \(V_g\) is calculated, the equivalent circuit can be completely solved to determine the capacitive VAr requirement of SPSEIG.

IV. EXPERIMENTAL SETUP AND MACHINE PARAMETERS

A six-phase induction generator which consists of two identical three-phase stator winding sets namely winding set \(abc\) (set-1) and winding set \(xyz\) (set-2) is utilized for conducting experiments. The necessary stator terminals have been taken out to the terminal box such that star or delta connection for both the sets can be realized. In this paper, the six stator phases are divided into two delta connected three-phase sets. The schematic diagram of SPSEIG with delta connected excitation capacitor bank and load is shown in Figure 3.

![Schematic diagram of the induction generator system employing a SPSEIG.](image)

The detailed winding scheme of the two stator winding sets \(abc\) and \(xyz\) of SPSEIG is given in Appendix – A (Figure 10). The ratings of the SPSEIG whose photograph is shown in Figure 11 of Appendix – B are as follows: 4-pole, delta connected stator, 2.2kW, 230V, 5A, 50Hz. The base values are

\[
\begin{align*}
V_{\text{base}} &= \text{rated phase voltage} = 230V \\
I_{\text{base}} &= \text{rated phase current} = 5/\sqrt{3} = 2.887A \\
Z_{\text{base}} &= V_{\text{base}}/I_{\text{base}} = 79.6675 \Omega \\
P_{\text{base}} &= V_{\text{base}} \times I_{\text{base}} = 0.664 \text{ kW} \\
N_{\text{base}} &= 1500 \text{ rpm} \\
f_{\text{base}} &= 50 \text{ Hz} \\
C_{\text{base}} &= 1 / (2\pi f_{\text{base}} Z_{\text{base}}) = 40 \mu\text{F}
\end{align*}
\]

The parameters of the equivalent circuit of the test machine, obtained from the results of the standard tests are
Magnetization characteristics of the machine play an important role in the analysis of SPSEIG. The magnetization curve (Figure 4) of the SPSEIG can be determined experimentally by driving the machine at rated synchronous speed and applying a variable voltage at rated frequency to the stator winding. The variation of $V_g/F$ with $X_M$ is non-linear due to magnetic saturation and the magnetization curve was incorporated in digital computer program for performance calculation using a piecewise linearization technique as given by Eq. (5) and (6).

\[
\begin{align*}
V_g/F & = 278.8 - 1.262X_M, & X_M < 70 \text{ ohms} \\
V_g/F & = 612 - 5.610X_M, & X_M \geq 70 \text{ ohms}
\end{align*}
\]

V. RESULTS AND DISCUSSION

It is observed that the SPSEIG is able to self-excite if proper value of shunt capacitor connected to either one of the three-phase winding set or both the winding sets. Moreover, the capacitance requirements vary depends upon the variation in speed and the required constant terminal voltage. The capacitance and VAr requirements to maintain constant terminal voltage for different constant speeds are computed for the following configurations.

- Capacitance and VAr requirements when both winding sets are excited and both are loaded equally.
- Capacitance and VAr requirements when both winding sets are excited and any one of the winding set is loaded.
- Capacitance and VAr requirements when any one of the winding set is excited and both winding sets are loaded equally.
- Capacitance and VAr requirements when any one of the winding set is excited and loaded.

All the above configurations are analyzed using the generalized mathematical model of SPSEIG given by Eq. (2) and GA based approach as discussed in Sections 2 and 3 respectively. For obtaining the capacitive VAr requirement to maintain the required terminal voltage under varying load conditions, solve for det $[Y]$ and find the unknown variables $X_{sh}$ and $F$ using the genetic algorithm method. After obtaining $X_{sh}$ and $F$, the equivalent circuit (Figure 1) is completely solved to determine the capacitive VAr requirement. The analysis is performed under three different speeds (below rated speed, rated speed, above rated speed). Moreover, for all the above cases, the same mathematical model (Eq. 2) can be used with modifications in few terms to consider the different capacitance and loading configurations.
5.1. Capacitance and VAr Requirements when Both Winding Sets are Excited and Both are Loaded Equally

In this configuration, both the winding sets abc and xyz of SPSEIG are connected with shunt capacitance. The SPSEIG is subjected to equal resistive loading on both winding sets abc and xyz. The analysis is carried out to find the range of values of capacitance and VAr requirement to maintain the rated terminal voltage of 1 p.u for three different constant speeds (0.95, 1.0, 1.05 p.u).

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The variations of shunt capacitance and VAr with output power to maintain the terminal voltage constant at 1 p.u at different constant speeds are shown in Figure 5a. The capacitance and capacitive VAr requirements increase as the speed decreases and as the output power increases. From Figure 5a, it is observed that a voltage regulator should be capable of providing a range of VAr from 2.0 to about 3.55 p.u. to maintain the terminal voltage at 1 p.u under varying speeds from 0.95 to 1.05 p.u with maximum output power of 3.3 p.u. The required shunt capacitance ranges from 0.6 to 1.3 p.u.

The variations of stator current with output power at constant terminal voltage are shown in Figure 5b. From this, it is observed that over a range of speed from 0.95 to 1.05 p.u, the stator currents are well below the rated value.

**Figure 5.** Variation of (a) VAr and Capacitance (b) Stator current with output power (Both the winding sets abc and xyz are excited and subjected to equal loading).
5.2. Capacitance and VAr Requirements when Both Winding Sets are Excited and Any One of the Winding Set is Loaded

For this configuration, both the winding sets $abc$ and $xyz$ are excited by shunt capacitance, but any one winding set (set $abc$) is subjected to resistive load. The shunt capacitance and VAr requirement to maintain rated terminal voltage under three different speeds are computed. Figure 6a shows the variation of capacitance and VAr with output power for varying speeds from 0.95 to 1.05 p.u. From Figure 6a, it is observed that the range of VAr and capacitance varies from 2.0 to about 3.8 p.u and 0.6 to 1.6 p.u respectively with maximum output power of 2.5 p.u. Figure 6b shows the variations of stator current with output power at constant terminal voltage. It is observed that the stator currents are well below the rated value.

![Figure 6a](attachment:image1.png)

**Figure 6a.** Variation of (a) VAr and Capacitance (b) Stator current with output power (Both the winding sets $abc$ and $xyz$ are excited and only winding set $abc$ is loaded).

5.3. Capacitance and VAr Requirements when Any One of the Winding Set is Excited and Both Winding Sets are Loaded Equally

For this type of configuration, only one set (set $abc$) is excited by shunt capacitance and the SPSEIG is subjected to equal resistive loading on both winding sets $abc$ and $xyz$. 

![Figure 6b](attachment:image2.png)
The variation of capacitance and VAr with output power to maintain terminal voltage at rated value of 1 p.u is shown in Figure 7a. From this, it is observed that the VAr and capacitance are varying from 3.8 to about 5.5 p.u and 1.1 to 2.0 p.u respectively under varying speeds from 0.95 to 1.05 p.u with maximum output power of 2.0 p.u. Figure 7b shows the variations of stator current with output power and the stator currents are found to be within rated value.

5.4. Capacitance and VAr Requirements when Any One of the Winding Set is Excited and Loaded

Here, winding set abc is alone excited and the same winding set is subjected to resistive loading with winding set xyz is kept open. The variations of shunt capacitance and VAr with output power are shown in Figure 8a. From this, it is found that VAr varies from 3.8 to about 5.3 p.u. to maintain the rated terminal voltage with maximum output power of 1.05 p.u. Also it is observed that the range of capacitance is 1.1 to 2.0 p.u. Figure 8b shows the variations of stator current with output power and it is observed that over a range of speed from 0.95 to 1.05 p.u, the stator currents are below the rated value.
From the above discussion, it is observed that the terminal voltage of SPSEIG depends on change in load, speed and shunt capacitance. The analytical and experimental study shows that the capacitance requirement varies from 0.6 to 1.6 p.u (24 to 64µF) when both winding sets are excited and 1.1 to 2 p.u (44 to 80µF) when one winding set is excited to maintain the rated terminal voltage of 1 p.u with speed varying from 0.95 to 1.05 p.u.

The results of all the configurations are summarised in Table 1.

Table 1: Comparative evaluation of variation of capacitance requirement and VAr requirement of SPSEIG from no load to full load under different modes of operation.

<table>
<thead>
<tr>
<th>Shunt Capacitance ( (C_{sh}) )</th>
<th>Loading Configuration</th>
<th>Speed ( (p.u) )</th>
<th>Capacitance Requirement ( (\mu F) )</th>
<th>VAr Requirement ( (p.u) )</th>
<th>Max. output power ( (p.u) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connected to both winding sets ( abc ) and ( xyz )</td>
<td>Both winding sets ( abc ) and ( xyz ) are subjected to equal loading</td>
<td>0.95</td>
<td>0.9 – 1.3</td>
<td>36 – 52</td>
<td>2.5-3.55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.0</td>
<td>0.8 – 1.2</td>
<td>32 – 48</td>
<td>2.3-3.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.05</td>
<td>0.6 – 0.8</td>
<td>24 – 32</td>
<td>2.0-2.75</td>
</tr>
<tr>
<td></td>
<td>Only winding set ( abc ) is subjected to load</td>
<td>0.95</td>
<td>0.9 – 1.6</td>
<td>36 – 64</td>
<td>2.6-3.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.0</td>
<td>0.8 – 1.3</td>
<td>32 – 52</td>
<td>2.38-3.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.05</td>
<td>0.6 – 1.0</td>
<td>24 – 40</td>
<td>2.0-3.3</td>
</tr>
<tr>
<td>Connected to winding set ( abc ) only</td>
<td>Both winding sets ( abc ) and ( xyz ) are subjected to equal loading</td>
<td>0.95</td>
<td>1.7 – 2.0</td>
<td>68 – 80</td>
<td>4.6-5.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.0</td>
<td>1.5 – 1.87</td>
<td>60 – 75</td>
<td>4.4-5.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.05</td>
<td>1.1 – 1.5</td>
<td>44 – 60</td>
<td>3.8-4.9</td>
</tr>
</tbody>
</table>

From the above discussion, it is observed that the terminal voltage of SPSEIG depends on change in load, speed and shunt capacitance. The analytical and experimental study shows that the capacitance requirement varies from 0.6 to 1.6 p.u (24 to 64µF) when both winding sets are excited and 1.1 to 2 p.u (44 to 80µF) when one winding set is excited to maintain the rated terminal voltage of 1 p.u with speed varying from 0.95 to 1.05 p.u. The results of all the configurations are summarised in Table 1.

Figure 8. Variation of (a) VAr and Capacitance (b) Stator current with output power (Only winding set \( abc \) is excited and subjected to load with winding set \( xyz \) is kept open).
subjected to load with winding set \(xyz\) is kept open

<table>
<thead>
<tr>
<th></th>
<th>1.0</th>
<th>1.4 – 1.8</th>
<th>56 – 72</th>
<th>4.37-5.3</th>
<th>1.05</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.05</td>
<td>1.1 – 1.4</td>
<td>44 – 56</td>
<td>3.8-4.9</td>
<td>1.6</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

In this paper, a simple and generalized mathematical model of SPSEIG based on nodal admittance method by inspection is developed for computer simulation to determine the necessary capacitance and VAr requirement to maintain constant terminal voltage for different constant speeds. The nodal admittance matrix can be formed from the equivalent circuit of SPSEIG directly by inspection and thus the proposed method completely avoids the lengthy mathematical derivations. Also, inspection based matrix equations are easier to modify such that inclusion or elimination of any equivalent circuit elements of SPSEIG can be carried out easily. Moreover, the proposed model can be utilised for the various configurations like shunt capacitance connected across (a) single three-phase winding set and (b) both the three-phase winding sets of SPSEIG. Good agreement between the analytical and experimental results is obtained which in general, verifies the accuracy of the proposed model and solution techniques. The analysis shows that the possibility of supplying two different and independent loads by using SPSEIG. Further, the operation can be continued even when one of the three-phase winding/excitation fails. Because, the remaining healthy generator winding still feeds the load with reduced power and it does not cause system shut down.

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APPENDIX - A

The six stator phases are divided into two three-phase winding sets (set-1: \(abc\) and set-2: \(xyz\)). The a, b and c phases of winding set-1 are displaced by 120° from each other. Similarly, x, y and z phases of winding set-2 are displaced by 120° from each other. The phase angle between winding set-1 and winding set-2 is 60° as shown in Figure 9. The Figure 9 shows that the x, y and z phases of winding set-2 are displaced by 180° with a, b and c phases of winding set-1 respectively. The complete winding diagram of SPSEIG is shown in Figure 10.

![Figure 9. Vector diagram of stator winding set-1 (abc) and winding set-2 (xyz).](image)

- Number of slots \((N_s)\) = 36
- Number of poles \((P)\) = 4
- Pole pitch = Number of slots / pole = 36/4 = 9
- Slot pitch angle = 360°\(N_s\) = 360°/36 = 10° (Mechanical)
  = 10° x \(P/2\) = 10° x 4/2 = 20° (Electrical)

**Winding set-1 (abc):**

- Starting slot of a-phase = 1st slot
Starting slot of $b$-phase $= (120^\circ/20^\circ) + 1 = 7^{th}$ slot
Starting slot of $c$-phase $= (240^\circ/20^\circ) + 1 = 13^{th}$ slot

Winding set-2 ($xyz$):
Starting slot of $x$-phase $= (180^\circ/20^\circ) + 1 = 10^{th}$ slot
Starting slot of $y$-phase $= (300^\circ/20^\circ) + 1 = 16^{th}$ slot
Starting slot of $z$-phase $= (60^\circ/20^\circ) + 1 = 4^{th}$ slot

**APPENDIX - B**

Figure 10. Winding diagram of SPSEIG (a) stator-1 (set $abc$) (b) stator-2 (set $xyz$).

Figure 11. Photograph of Six-phase self-excited induction generator (SPSEIG) and shunt excitation capacitor bank.

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